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Slides: [STAT430](https://docs.google.com/presentation/d/1AQQ5cMRcRopuNHrSmrsZV4Ev73ihtq2ObKCE2-U_8Sk/edit?usp=sharing)

Introduction**:**

* Model uses real-world instances obtained from Amsterdam University Medical Centers and artificial instances ( generated from parameterized instance generator)

**Title:** *Integrated patient-to-room and nurse-to-patient assignment in hospital wards*

**Citation:** Brandt, Tabea, et al. "Integrated patient-to-room and nurse-to-patient assignment in hospital wards." arXiv preprint arXiv:2309.10739 (2023). <https://research.utwente.nl/en/publications/integrated-patient-to-room-and-nurse-to-patient-assignment-in-hos>

Motivation:

* **General overview of problem** 
  + Mixed integer program (MIP) and an efficient heuristic

* **Why problem/model is important**
  + This model is useful in improving how well a hospital operates
  + Increases employee and patient satisfaction, quality of care, and hospital efficiency
* **Integration Problem(why model is important)**
  + While single-resource optimizations or single-goal problems can be helpful, integrated planning of multiple resources will help improve efficiency
  + PRA: Patient-to-room assignment
  + NPA: Nurse-to-patient assignment
  + This paper explicitly considers decisions on PRAs and NPAs in one integrated optimization problem for the first time
  + Allows for additional insight based on the interaction between PRA & NPA
* **Background info to understand problem formulation**

Problem Presentation:

* **Goals (Why pursue this optimization problem):** 
  + Determining PRA (patient to room assignment) to improve resource utilization (ex:hospital beds) based on patient needs and preferences
  + Determining NPA (nurse-to-patient assignment) to increase employee satisfaction
  + Employes new heterogeneity check between patient admission and discharge times for room assignment
  + increase employee satisfaction
  + efficient operations of a hospital
  + quality of care
  + Optimizing nurses walking distances between patients
  + Optimize walking distances for nurses to help minimize negative effects (ex:spread of infection, disturbance of patients)
* **Stipulations:** 
  + Patients are always admitted and discharged in the morning between a night shift and the following early shift
  + Patient transfers between rooms are possible and are assumed to take place at most once a day for each patient between a night shift and early shift
  + Gender-mixed rooms should be avoided if possible
  + Large age difference between patients in the same room should be minimized
  + Nurses will work at most one shift per day
  + Fair distribution of workload for staff

Model Presentation:

* **Input parameters and Sets**
  + **Sets**
    - P - patients (set)
    - F - female patients (subset)
      * Subset of all patients P
    - M - male patients (subset)
      * Subset of all patients P
    - N - nurses (set)
    - R - rooms (set)
    - A - additional rooms (set)
      * This includes the nursing station
      * Note the intersection of R & A is Null
    - S - shifts (set)
      * {1, …, S}
    - S\_early - early shifts (subset)
      * S\_early = {1, 4, 7, …, } ∊ S
    - S\_late - late shifts (subset)
      * ∊ S
    - S\_night - night shifts (subset)
      * ∊ S
    - S(n) - shifts that nurse n ∊ N is assigned to (subset)
    - E - possible equipment types in rooms
    - E(r) - equipment types present in room r (subset)
      * ∊ E
    - E(p,s) - desired equipment types of patient p during shift s (subset)
      * ∊ E
    - L = {1,2,3} - possible skill levels of nurses (1= trainee, 2=regular, 3= experienced) (subset)
    - N\_prev(p) - nurses that patient p ∊ P has already been assigned to during at least one shift in a previous planning period (subset)

|  |  |
| --- | --- |
| Set | Description |
| P | Set of Patients |
| F | Subset of Female Patients |
| M | Subset of Male Patients |
| N | Set of Nurses |
| Nprev(p) | Subset of Nurses Patient p ∊ P has Already Been Assigned to at Least 1 Shift in a Previous Planning Period |
| R | Set of Rooms |
| A | Set of Additional Rooms |
| S | Set of Shifts {1,2,...,S} |
| Searly | Subset of Early Shifts |
| Slate | Subset of Late Shifts |
| Snight | Subset of Night Shifts |
| S(n) | Subset of shifts that n ∊ N is assigned to |
| E | Set of Possible Equipment Type in the Rooms |
| E(r) | Subset of Equipment Types Avilable in room r ∊ R |
| E(p,s) | Subset of Desired Equipment Types of patient p ∊ P during shift s ∊ S |
| L | Set of Nurse Skill Levels {1,2,3} |

* + **Parameters** 
    - ad\_shift(p)
      * admission shift s, first shift of patient’s stay on the ward (Patients are always admitted and discharged in the morning between a night shift and the following early shift)
      * ∊ S\_early
      * Value set to 0 if patient has already been on the ward during the last shift of the previous planning period
    - di\_shift(p)
      * Discharge shift s, last shift of patient’s stay on the ward
      * ∊ S\_night
      * Value set to S+1 if patient will still be on ward in next planning period
    - y\_prev(p)
      * r ∊ R that p ∊ P with ad\_shift(p)=0 has been assigned to during the last shift of the previous planning period
    - num\_beds(r)
      * Number of beds likely in {1,2,3,4} in room r ∊ R
    - age\_group(p)
      * Age group of patient (grouped by 10 years)
    - skill\_level(n)
      * Skill of nurse n; values are 1,2,3
    - skill\_req(p,s)
      * Skill level required by p ∊ P during shift s ∊ S \ S\_night (all shifts between admission and discharge); values are 0, 1, 2
    - w\_load(p,s)
      * Workload from taking care of p ∊ P during s ∊ S; a non-negative number
      * This values depends on the age group of the patient, specific condition, time since admission, and whether it is a day or night shift.
    - max\_load(n,s)
      * Non-negative number specifying the max workload allowed for n ∊ N during s ∊ S
    - dis(r,r’)
      * Walking distance between r, r’ ∊ R
    - dist(a,r)
      * Walking distance between a ∊ A and r ∊ R
    - walk\_pat°(s)
      * Non-negative weight of the walking patterns depending on shift s ∊ S
      * A high value indicates that most nurse walk directly from patient to patient during shift s (circular pattern)
    - walk\_pat\*(s)
      * nonnegative weight for different walking patterns depending on the shift s ∈ S. A high value indicates that most nurses walk directly from additional rooms such as the nursing station to patient rooms and back during shift s (star-like pattern)

| Parameter | Description |
| --- | --- |
| ad\_shift(p) | Shift s ∈ Searly during which patient p ∈ P is admitted. The value is set to 0 if patient p has already been on the ward during the last shift of the previous planning period |
| dis\_shift(p) | Shift s ∈ Slate during which patient p ∈ P is discharged. The value is set to S + 1 if patient p will still be on the ward after the last shift S of the planning period |
| yprev(p) | room r ∈ R that patient p ∈ P with ad\_shift(p) = 0 has been assigned to during the last shift of the previous planning period |
| num\_beds(r) | Non-negative integer specifying the number of beds in room r ∈ R |
| age\_group(p) | Age group of patient p ∈ P computed as age group(p) = floor(age(p)/10) with age(p) denoting the age of the patient in years |
| skill\_level(n) | Skill level of nurse n ∈ N |
| skill\_req(p,s) | Minimum skill level of a nurse required by patient p ∈ P during shift s ∈ S \ Snight |
| w\_load(p,s) | Non-negative number specifying the workload resulting from taking care of patient p ∈ P during shift s ∈ S |
| max\_load(n,s) | Non-negative number specifying the maximum workload allowed for nurse n ∈ N during shift s ∈ S(n) |
| dist(r, r’) | Non-negative number specifying the walking distance between rooms r, r′ ∈ R |
| dist(a,r) | Non-negative number specifying the walking distance between additional room a ∈ A and room r ∈ R |
| walk\_pato(s) | Non-negative weight for different walking patterns depending on the shift s ∈ S. A high value of walk\_pato(s) indicates that most nurses walk directly from patient to patient during shift s (circular pattern) |
| walk\_pat\*(s) | Non-negative weight for different walking patterns depending on the shift s ∈ S. A high value of walk\_pat\*(s) indicates that most nurses walk directly from additional rooms such as the nursing station to patient rooms and back during shift s (star-like pattern) |

* **Decision Variables** 
  + Y\_p\_r\_s
    - Binary variable indicating whether patient p ∈ P is assigned to room r ∈ R during early shift s ∈ S\_early (only defined if ad\_shift(p) <= s <= di\_shift(p), i.e., if patient p is on the ward during early shift s)
  + F\_in\_room\_r,s
    - Binary variable indicating whether at least one female patient is assigned to room r ∈ R during early shift s ∈ S\_early
  + M\_in\_room\_r,s
    - Binary variable indicating whether at least one male patient is assigned to room r ∈ R during early shift s ∈ S\_early
  + vio^gender\_r,s
    - Binary variable indicating whether more than one gender is accommodated to room r ∈ R during early shift s ∈ S\_early
  + Trans\_p,s
    - Binary variable indicating whether patient p ∈ P is transferred to a different room after night shift s ∈ (S\_night U {0}) \ {S} (and before early shift s+1
  + age\_group^max\_r,s
    - Nonnegative fractional variable representing the maximum age group among all patients p ∈ assigned to room r ∈ R during early shift s ∈ S\_early
  + age\_group^min\_r,s
    - Nonnegative fractional variable representing the minimum age group among all patients p ∈ assigned to room r ∈ R during early shift s ∈ S\_early
  + X\_p,n,s
    - Binary variable indicating whether patient p ∈ P is assigned to nurse n ∈ N during shift s ∈ S
    - (only defined if s ∈ S(n) and ad\_shift(p) ≤ s ≤ di\_shift(p), i.e., if nurse n is assigned to shift s and patient p is on the ward during shift s)
  + vio^skill\_p,s
    - binary variable indicating whether patient p ∈ P is assigned to a nurse with a lower skill level than required during shift s ∈ S \ S\_night
    - (only defined if ad\_shift(p) ≤ s ≤ di\_shift(p) and skill req(p, s) ≥ 2, i.e., if patient p is on the ward during shift s and requires at least an experienced nurse during shift s)
  + Ever\_assigned\_p,s
    - binary variable indicating whether patient p ∈ P is assigned to nurse n ∈ N during at least one shift s ∈ S
  + vio^load\_n,s
    - nonnegative fractional variable representing the excess load assigned to nurse n ∈ N during shift s ∈ S
    - (only defined if s ∈ S(n), i.e., if nurse n is assigned to shift s
  + vio^fair\_n,n’,s
    - nonnegative fractional variable representing the excess in relative workload (relative to the desired maximum) of nurse n ∈ N compared to nurse n ′ ∈ N during shift s ∈ S
  + vio^fair\_n,n’
    - nonnegative fractional variable representing the overall excess in relative workload (relative to the desired maximum) of nurse n ∈ N compared to nurse n ′ ∈ N
  + In\_room\_n,r,s
    - binary variable indicating whether nurse n ∈ N is assigned at least one patient in room r ∈ R during shift s ∈ S
    - (only defined if s ∈ S(n), i.e., if nurse n is assigned to shift s)
  + Dist\_n,s
    - nonnegative fractional variable representing the total walking distance for nurse n ∈ N during shift s ∈ S (only defined if s ∈ S(n), i.e., if nurse n is assigned to shift s)
  + Both\_rooms\_n,r,r’,s
    - binary variable indicating whether nurse n ∈ N is assigned patients in both room r ∈ R and room r′ ∈ R during shift s ∈ S
    - (only defined if assign(n, s) = 1, i.e., if nurse n is assigned to shift s)

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| --- | --- |
| Variable | Description |
|  | Binary variable indicating whether patient p ∈ P is assigned to room r ∈ R during early shift s ∈ Searly |
| f\_in\_roomr,s | Binary variable indicating whether at least one female patient is assigned to room r ∈ R during early shift s ∈ Searly |
| m\_in\_roomr,s | Binary variable indicating whether at least one male patient is assigned to room r ∈ R during early shift s ∈ Searly |
| vior,sgender | Binary variable indicating whether more than one gender is accommodated in room r ∈ R during shift s ∈ Searly |
| transp,s | Binary variable indicating whether patient p ∈ P is transferred to a different room after night shift s ∈ (Snight ∪ {0}) \ {S} |
| age\_groupr,smax | Non-negative fractional variable representing the maximum age group among all patients p ∈ P assigned to room r ∈ R during early shift s ∈ Searly |
| age\_groupr,smin | Non-negative fractional variable representing the minimum age group among all patients p ∈ P assigned to room r ∈ R during early shift s ∈ Searly |
| xp,n,s | Binary variable indicating whether patient p ∈ P is assigned to nurse n ∈ N during shift s ∈ S |
| viop,sskill | Binary variable indicating whether patient p ∈ P is assigned to a nurse with a lower skill level than required during shift s ∈ S \ Snight |
| ever\_assignedp,n | Binary variable indicating whether patient p ∈ P is assigned to nurse n ∈ N during at least one shift s ∈ S |
| vion,sload | Non-negative fractional variable representing the excess load assigned to nurse n ∈ N during shift s ∈ S |
| vion,n’,sfair | Non-negative fractional variable representing the excess in relative workload of nurse n ∈ N compared to nurse n’ ∈ N during shift s ∈ S |
| vion,n’fair | Non-negative fractional variable representing the overall excess in relative workload of nurse n ∈ N compared to nurse n’ ∈ N |
| in\_roomn,r,s | Binary variable indicating whether nurse n ∈ N is assigned at least one patient in room r ∈ R during shift s ∈ S |
| distn,s | Non-negative fractional variable representing the total walking distance for nurse n ∈ N during shift s ∈ S |
| both\_roomsn,r,r’,s | Binary variable indicating whether nurse n ∈ N is assigned patients in both room r ∈ R and room r’ ∈ R during shift s ∈ S |

* **Model**
  + IPRNPA: Integrated patient-to-room and nurse-to-patient assignment problem
  + The integration of PRA & NPA is at the ward level over a given planning period (typically one or several weeks)
  + All information about the patients (including admission and discharge times) are known at the beginning of the planning period
* **Objective Function** 
  + Weighted sum of 8 objectives
  + PRA: 1-4
  + NPA: 5-6
  + Interaction between problems: 7-8

1. Patient Transfers Objective
   1. Minimize the number of patient transfers across all patients and shifts
   2. Minimize ∑ trans\_p\_s
   3. Summation:
      1. p ∊ P (all patients)
      2. s ∊ S\_night \ {S}: ad\_shift(p) <= s <= di\_shift(p) - 1 (for patient p, sum over all shifts the patient is in the ward, exclude final shift S)
2. Patient Inconvenience Objective
   1. Minimize age group difference across all rooms and shifts
   2. Minimize ∑ (age\_group^max\_r,s - age\_group^min\_r,s)
   3. Summation:
      1. r ∊ R (all rooms)
      2. s ∊ S\_early (all Early Shifts)
3. Gender Mixing Objective
   1. Minimize gender mixing across all rooms and shifts
   2. Minimize ∑ vio^gender\_r,s
   3. Summation:
      1. r ∊ R (all rooms)
      2. s ∊ S\_early (all Early Shifts)
4. Equipment Violation Objective
   1. Minimize required equipment violations across all rooms and shifts
   2. Minimize ∑ y\_p\_r\_s
   3. Summation:
      1. p ∊ P (all patients)
      2. r ∊ R (all rooms)
      3. s ∊ S\_early: ad\_shift(p) <= s <= di\_shift(p) (all Early Shifts during time in ward)
      4. E(p,s) \ E(r) =/= Null (desire equipment for patient p during shift s less the equipment types in room r is not zero – violation of needed equipment)
5. Continuity of care Objective
   1. Minimize the number of different nurses that treat each patient across all patients
   2. Minimize ∑ ever\_assigned\_p,n
   3. Summation:
      1. p ∊ P (all patients)
      2. n ∊ N \ N\_prev(p) (our concern is with new assignment in this timeframe so it is all Nurses less Nurses previously assigned to)
6. Penalization of skill level requirements and undesired workload distribution Objective
   1. Minimization of violations of skill level requirements of patients and undesired workload distributions for nurses
   2. Minimize the following 4 Summations:
      1. ~
      2. ~
      3. ~
      4. ~
7. Assigning the minimum number of nurses per room Objective
   1. Minimize number of nurses assigned to rooms across all shifts
   2. Minimize ∑ in\_room\_n,r,s
   3. Summation:
      1. n ∊ N (all nurses)
      2. r ∊ R (all rooms)
      3. s ∊ S (all shifts)
8. Walking distances Objective
   1. Minimize walking distances across all nurses and shifts
   2. Minimize ∑ dist\_n,s
   3. Summation:
      1. n ∊ N (all nurses)
      2. s ∊ S(n) (all shifts) (nurse must be assigned to that shift)

* Explain/simplify objective function
* **Types of constraints (can choose 8 to present in presentation). Note: this is probably the most important part of the presentation since this section is worth 14 points**

1. Assignment of patients to rooms
   1. Each patient p ∊ P is assigned to exactly one room r ∊ R during each early shift s ∊ S\_early between admission and discharge
   2. No room r ∊ R can be assigned more than num\_beds(r) patients during any early shift s ∊ S\_early
   3. Room gender assignments
   4. No room r ∊ R should be assigned both female and male patients during any early shift s ∊ S\_early
2. Patients Transfers
3. Inconvenience of Patients
4. Assignment of Patients to Nurses
5. Workload of Nurses
6. Assignment of all patients in the same room to the same nurse
7. Walking distance of nurses

Numerical Analysis

* **Practical Application:** 
  + Parameterized instance generator that generates large instances of realistic test cases. The generator can generate large instances of test cases based on user defined parameters
  + Real world instances from a Short Stay Unit of the research partnered hospital.
    - Admission/discharge dates known in advance, ward is closed on weekends
    - Nurses work on three shifts with two different levels(experienced and trainee)
    - 17 patient room of varying sizes(single, double, triple, quadruple)
    - Generator for the missing data
* **Metric of Success(how they chose to evaluate the performance):** 
  + Artificial dataset:
    - using the generator to create artificial datasets based on different scenarios.
    - Evaluate the performance of the three methods(MIP, sequential model approach, heuristic approach) based on the resulting objective values as percentages
    - Evaluate based on running time and objective value
  + Real-world dataset:
    - Evaluate based on running time and objective value

Conclusion/Recommendation

* The heuristic solution is better than the standard in terms of seeking the right balance between running time efficiency and optimal solution.
  + Ideal for real world healthcare settings
* Devised the parameterized instance generator
* Extension: dynamic changes with high degrees of uncertainty, rostering decisions

Supplementary Material: [STAT 430 Infeasible Solutions - Mid-Sem Project](https://docs.google.com/document/d/1BBELOme7AWo-vw9KdqWsmTT0NdTsb5jEMeBCqdYuaLQ/edit?usp=sharing)

* Infeasible Scenarios
  + For each type of constraint in model (only for up to 10 of them)

Constraint 1:

Each patient is is assigned to exactly one room during each early shift between their admission and their discharge:

Scenario 1:

The patient is assigned to two rooms during early shift .

Constraint 2:

No room r ∈ R can be assigned more than num\_beds(r) patients during any early shift s ∈ S early:

Scenario 2:

Room r ∈ R is assigned more than num\_beds(r) during early shift s ∈ S:

Constraint 3:

The variable () is set to one if at least one female (male) patient is assigned to room r during early shift :

Scenario 3:

One female (male) patient is assigned to room r during early shift but

the variable () is not set to one(zero):

Constraint 4:

No room r ∈ R should be assigned both female and male patients during any early shift :

: binary variable indicating whether more than one gender is accommodated in room r ∈ R during shift

Scenario 4:

A room r ∈ R gets assigned both female and male patients during any early shift when the is marked as 0:

Constraint 5:

The patient transfer variables are set correctly for each patient p ∈ P and each night shift night between their admission and discharge:

: binary variable indicating whether patient p ∈ P is transferred to a different room after night shift s ∈ ( ∪ {0}) \ {S} (and before early shift s + 1) (only defined if ad\_shift(p) ≤ s ≤ di\_shift(p) − 1)

Scenario 5:

The patient p ∈ P is transferred to a different room after night shift s ∈ ( ∪ {0}) \ {S}(and before early shift s + 1) but the patient transfer variables is set incorrectly:

Constraint 6:

The patient transfer variables that indicate a transfer between the last shift of the previous planning period (shift 0) and the first (early) shift of the current planning period (shift 1) are set correctly for each patient p ∈ P with ad\_shift(p) = 0:

Scenario 6:

A patient p ∈ P transfer between the last shift of the previous planning period (shift 0) and the first (early) shift of the current planning period (shift 1) is set incorrectly in the variable :

Constraint 7:

The variable is restricted by the maximum age group of patients in room r ∈ R during early shift :

age\_ group(p): age group of patient p ∈ P computed as with denoting the age of the patient in years

: nonnegative fractional variable representing the maximum age group among all patients p ∈ P assigned to room r ∈ R during early shift

Scenario 7:

The maximum age group of patients(ex: 9) in room r ∈ R during early shift is higher than than the variable (ex: 8):

Constraint 8:

The variable is restricted by the minimum age group of patients in room r ∈ R during early shift :

: nonnegative fractional variable representing the minimum age group among all patients p ∈ P assigned to room r ∈ R during early shift :

Note that the coefficient 12 on the right-hand side needs to be increased if patients with ages 130 or older (age group 13 or higher) are present

Scenario 8:

The minimum age group of patient(ex: 6) in room r ∈ R during early shift is higher than than the expression (ex: 4) :

Constraint 9:

The variable age is set to zero if no patients are assigned to room r ∈ R during early shift :

Note that the coefficient 12 on the right-hand side needs to be increased if patients with ages 130 or older (age group 13 or higher) are present.

Scenario 9:

No patients are assigned to room r ∈ R during early shift , is set to a non-zero number(ex: 1):

Constraint 10:

The value of the variable must not be smaller than the value of the variable for any room r ∈ R and any early shift :

Scenario 10:

A one of the room r ∈ R and in an early shift , the value of the variable (ex: 5) becomes smaller than the value of the variable (ex: 6).